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QUADRIMPULSE FLYBACK WITH REVERSION

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16. Abstract
Quadrimpulse flyback between two points which move along round coplanar orbits uniformly in the same direction is noted. Two pulses are applied in flyback from the first point to the second; two are applied in reverse flyback. A Homann flyback is one whose orbits "there" and "back" are Homann semi-ellipses. The orbit for which δF is smallest is the optimal orbit.

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QUADRIMPULSE FLYBACK WITH REVERSION

G. V. Ufimtsev

Quadrimpulse flyback between two points which move along /41 round coplanar orbits uniformly in the same direction is examined. Two pulses are applied in flyback from the first point to the second, two--in reverse flyback. A round orbit, in which the first /42 pulse is applied, will be called the starting orbit; the orbit which is the target of our flight--the destination orbit. Let us seek the energetically optimal flyback, considering the fact that the total flyback time is restricted, and the stay time in the destination orbit must not be less than some quantity.

Let a_1 be the radius of the starting orbit; a_2--the radius of the destination orbit; p_1, e_1, ω_1--Keplerian elements of the flyback ellipse from the starting orbit to the destination orbit. p_2, e_2, ω_2--elements of the ellipse of reverse flyback; k--constant of gravitation of the central field; n_1--mean motion of a point along the starting orbit; n_2--mean motion of a point along the destination orbit; $t_1, \ldots, t_4$--moments of pulse application; u_{10}, u_{20}--initial phases of motion of the first and second points between which flyback is effected, corresponding to time t_0; v_{ii}, v_{iu}, i = 1, 2--true anomalies in ellipses of flyback at instant of pulse application.

Then the boundary conditions can be written in the form of the following equations:

$$
\begin{gathered}
p_1 - a_1 (1 + e_1 \cos v_{1\text{н}}) = 0, \\
p_1 - a_2 (1 + e_1 \cos v_{1\text{к}}) = 0, \\
p_2 - a_2 (1 + e_2 \cos v_{2\text{н}}) = 0, \\
p_2 - a_1 (1 + e_2 \cos v_{2\text{к}}) = 0, \\
u_{10} + n_1 (t_1 - t_0) - \omega_1 - v_{1\text{н}} = 0, \\
u_{20} + n_2 (t_2 - t_0) - \omega_1 - v_{1\text{к}} - 2\pi l_1 = 0, \\
u_{20} + n_2 (t_3 - t_0) - \omega_2 - v_{2\text{н}} = 0, \\
u_{10} + n_1 (t_4 - t_0) - \omega_2 - v_{2\text{к}} - 2\pi l_2 = 0, \\
\varphi (p_1, e_1, v_{1\text{н}}, v_{1\text{к}}) - k (t_2 - t_1) = 0, \\
\varphi (p_2, e_2, v_{2\text{н}}, v_{2\text{к}}) - k (t_4 - t_3) = 0,
\end{gathered}
$$

where l_1, l_2--whole numbers which will be defined later, and function ϕ has the form

$$\varphi(p,\ e,\ v_{\text{н}},\ v_{\text{к}})=p^{\frac{3}{2}}\int_{v_{\text{н}}}^{v_{\text{к}}}(1+e\cos v)^{-2}\,dv. \tag{2}$$

If T_0--maximum time of entire flyback, and T_s--minimum time of stay in destination orbit, then limitations on time can be written in the following manner:

$$\begin{aligned} T_0-(t_4-t_1)-\tau_0^2=0,\\ T_{\text{п}}-(t_3-t_2)+\tau_{\text{п}}^2=0; \end{aligned} \tag{3}$$

here τ_0 and τ_s are unknown real numbers.

Let us designate pulses in the order of their sequence by /43
ΔV_i, i = 1, 2, 3, 4. Then

$$\begin{aligned} \Delta V_1&=V(a_1,\ e_1,\ p_1),\\ \Delta V_2&=V(a_2,\ e_1,\ p_1),\\ \Delta V_3&=V(a_2,\ e_2,\ p_2),\\ \Delta V_4&=V(a_1,\ e_2,\ p_2),\\ V(a,\ e,\ p)&=k\left(\frac{3}{a}-\frac{1-e^2}{p}-2\frac{\sqrt{p}}{a^{3/2}}\right). \end{aligned} \tag{4}$$

The function V provides expression for the pulse in transfer from a round orbit to an elliptical one and vice versa. The angle between the direction of the pulse and the direction of the initial velocity can be any size.

Let us minimize some function F, which reflects energy losses in effecting flyback, which we will consider symmetrical with respect to the variables $\alpha_i \Delta V_i$, where α_i--weights with which are selected the appropriate pulses. These weights can denote, e.g., the portion of the pulse in which fuel is expended. If a portion of the pulse required for transfer from orbit to orbit is extinguished by the resistive medium, then the corresponding $\alpha_i < 1$. Moreover, we will consider that function F depends on flyback time $(t_4 - t_1)$ so that

$$\frac{\partial F}{\partial t_4} = -\frac{\partial F}{\partial t_1}. \tag{5}$$

The formulated problem is a problem of searching a conventional extremum of function F. Let us introduce the undefined coefficients $\lambda_1, \lambda_2, \ldots, \lambda_8$ for the first eight boundary conditions (1), μ_1, μ_2--for the two last boundary conditions of (1), and ν_1, ν_2--for limitations on time (3) and form function Φ of the following form:

$$\Phi = F + \sum_{i=1}^{8} \lambda_i L_i + \sum_{j=1}^{2} (\mu_j M_j + \nu_j N_j), \tag{6}$$

where L_i, M_j, N_j are the left parts of equations (1) and (3). Then the necessary conditions of the minimum of function F are written as follows:

$$\frac{\partial \Phi}{\partial y_l} = 0, \quad l = 1, 2, \ldots, 16, \tag{7}$$

where

$$\bar{y} = (p_1, e_1, \omega_1, v_{1н}, v_{1к}, p_2, e_2, \omega_2, v_{2н}, v_{2к}, t_1, t_2, t_3, t_4, \tau_0, \tau_п).$$

As a result, we derive a system of equations (1), (3), (7) consisting of 28 equations with 28 unknowns.

Let us write the necessary conditions for the minimum in a more unfolded form /44

$$
\begin{aligned}
\frac{\partial F}{\partial p_1}+\lambda_1+\lambda_2+\mu_1\frac{\partial \varphi_1}{\partial p_1}&=0,\\
\frac{\partial F}{\partial e_1}-\lambda_1 a_1\cos v_{1\text{н}}-\lambda_2 a_2\cos v_{1\text{к}}+\mu_1\frac{\partial \varphi_1}{\partial e_1}&=0,\\
-\lambda_5-\lambda_6&=0,\\
\lambda_1 a_1 e_1\sin v_{1\text{н}}-\lambda_5-\mu_1\frac{a_1^2}{\sqrt{p_1}}&=0,\\
\lambda_2 a_2 e_1\sin v_{1\text{к}}-\lambda_6-\mu_1\frac{a_2^2}{\sqrt{p_1}}&=0,\\
\frac{\partial F}{\partial p_2}+\lambda_3+\lambda_4+\mu_2\frac{\partial \varphi_2}{\partial p_2}&=0,\\
\frac{\partial F}{\partial e_2}-\lambda_3 a_2\cos v_{2\text{н}}-\lambda_4 a_1\cos v_{2\text{к}}+\mu_2\frac{\partial \varphi_2}{\partial e_2}&=0,\\
-\lambda_7-\lambda_8&=0,\\
\lambda_3 a_2 e_2\sin v_{2\text{н}}-\lambda_7-\mu_2\frac{a_2^2}{\sqrt{p_2}}&=0,\\
\lambda_4 a_1 e_2\sin v_{2\text{к}}-\lambda_8+\mu_2\frac{a_1^2}{\sqrt{p_2}}&=0,\\
\frac{\partial F}{\partial t_1}+\nu_1+\mu_1 k+\lambda_5 n_1&=0,\\
-\nu_2-\mu_1 k+\lambda_6 n_2&=0,\\
\nu_2+\mu_2 k+\lambda_7 n_2&=0,\\
\frac{\partial F}{\partial t_4}-\nu_1-\mu_2 k+\lambda_8 n_1&=0,\\
-2\nu_1\tau_0&=0,\\
-2\nu_2\tau_{\text{п}}&=0.
\end{aligned}
\tag{8}
$$

This implies

$$\frac{\partial F}{\partial p_1}=\frac{\partial F}{\partial \Delta V_1}\frac{\partial \Delta V_1}{\partial p_1}+\frac{\partial F}{\partial \Delta V_2}\frac{\partial \Delta V_2}{\partial p_1}$$ etc.

and also

$$\varphi_i=\varphi(p_i,\ e_i,\ v_{i\text{н}},\ v_{i\text{к}}),\ i=1,\ 2.$$

Using the properties of function F(5), from equations (8) we can derive

$$
\left.
\begin{aligned}
\lambda_6 &= -\lambda_5, \\
\lambda_7 &= \lambda_5, \\
\lambda_8 &= -\lambda_5, \\
\mu_2 &= \mu_1, \\
\nu_1 &= -\frac{\partial F}{\partial t_1} - n_1\lambda_5 - k\mu_1, \\
\nu_2 &= -n_2\lambda_5 - k\mu_1.
\end{aligned}
\right| \qquad (9)
$$

The first, second, sixth and seventh equations of the group /45
of equations (8) can be solved with respect to the undefined coefficients λ_1, λ_2, λ_3, λ_4. Consequently, we will find that

$$
\left.
\begin{aligned}
&\lambda_1 = -\frac{A_1}{\Delta_1}, \quad \lambda_2 = \frac{A_2}{\Delta_1}, \quad \lambda_3 = -\frac{A_3}{\Delta_2}, \quad \lambda_4 = \frac{A_4}{\Delta_2}, \\
&A_1 = \frac{\partial F}{\partial p_1} a_2 \cos v_{1\text{к}} + \frac{\partial F}{\partial e_1} + \mu_1\left(\frac{\partial \varphi_1}{\partial p_1} a_2 \cos v_{1\text{к}} + \frac{\partial \varphi_1}{\partial e_1}\right), \\
&A_2 = \frac{\partial F}{\partial p_1} a_1 \cos v_{1\text{н}} + \frac{\partial F}{\partial e_1} + \mu_1\left(\frac{\partial \varphi_1}{\partial p_1} a_1 \cos v_{1\text{н}} + \frac{\partial \varphi_1}{\partial e_1}\right), \\
&A_3 = \frac{\partial F}{\partial p_2} a_1 \cos v_{2\text{к}} + \frac{\partial F}{\partial e_2} + \mu_1\left(\frac{\partial \varphi_2}{\partial p_2} a_1 \cos v_{2\text{к}} + \frac{\partial \varphi_2}{\partial e_2}\right), \\
&A_4 = \frac{\partial F}{\partial p_2} a_2 \cos v_{2\text{н}} + \frac{\partial F}{\partial e_2} + \mu_1\left(\frac{\partial \varphi_2}{\partial p_2} a_2 \cos v_{2\text{н}} + \frac{\partial \varphi_2}{\partial e_2}\right), \\
&\Delta_1 = a_2 \cos v_{1\text{к}} - a_1 \cos v_{1\text{н}}, \\
&\Delta_2 = a_1 \cos v_{2\text{к}} - a_2 \cos v_{2\text{н}}.
\end{aligned}
\right| \qquad (10)
$$

We should note that this solution is always possible since $\Delta_1 \neq 0$, $\Delta_2 \neq 0$. Let us assume the contrary. Let, for example, $_1 = 0$; from the first two equations of (1) it follows that $\alpha_1 = \alpha_2$. We will not discuss such cases.

Symmetrical Flybacks

Let us take as the origin of the time count $\frac{1}{2}(t_1 + t_4)$. Then

$$
-t_4 = t_1. \qquad (11)
$$

We will call the flyback symmetrical, if the following equalities are fulfilled:

$$\left.\begin{aligned} P_2 &= P_1 \equiv P, \\ e_2 &= e_1 \equiv e, \\ -v_{2\text{к}} &= v_{1\text{н}} \equiv v_{\text{н}}, \\ -v_{2\text{н}} &= v_{1\text{к}} \equiv v_{\text{к}}, \\ -t_3 &= t_2. \end{aligned}\right\} \tag{12}$$

Using formulas (2), (4) and the property of symmetricity of function F with respect to pulses ΔV_i (all α_i we assume equal to one), it is easy to show the validity of identities

$$\left.\begin{gathered} \frac{\partial\varphi_1}{\partial p_1} \equiv \frac{\partial\varphi_2}{\partial p_2} \equiv \frac{\partial\varphi}{\partial p}, \quad \frac{\partial\varphi_1}{\partial e_1} \equiv \frac{\partial\varphi_2}{\partial e_2} \equiv \frac{\partial\varphi}{\partial e}, \\ \Delta V_1 \equiv \Delta V_4, \qquad \Delta V_2 \equiv \Delta V_3, \\ \frac{\partial F}{\partial p_1} \equiv \frac{\partial F}{\partial p_2} \equiv \frac{1}{2}\frac{\partial F}{\partial p}, \qquad \frac{\partial F}{\partial e_1} \equiv \frac{\partial F}{\partial e_2} \equiv \frac{1}{2}\frac{\partial F}{\partial e}. \end{gathered}\right\} \tag{13}$$

Using the identities in (13) for equations (10), we find that

$$\left.\begin{gathered} A_1 = A_4, \qquad A_2 = A_3, \qquad \Delta_1 = -\Delta_2 = \Delta, \\ \lambda_1 = \lambda_4, \qquad \lambda_2 = \lambda_3. \end{gathered}\right\} \tag{14}$$

Now, the necessary conditions of the minimum (8), using equations (9), (10) and (14), can be reduced to five equations: /46

$$\left.\begin{gathered} \mu = \frac{1}{2}\left[\frac{\partial F}{\partial p} a_1 a_2 \sin(v_{\text{н}} - v_{\text{к}}) + \frac{\partial F}{\partial e}(a_1 \sin v_{\text{н}} - a_2 \sin v_{\text{к}})\right], \\ \lambda = \frac{a_1 a_2}{2}\left\{\frac{\partial F}{\partial p}\left[\frac{1}{\sqrt{p}}(a_1^2 \sin v_{\text{к}} \cos v_{\text{н}} - a_2^2 \sin v_{\text{н}} \cos v_{\text{к}}) - \right.\right. \\ \left. - \frac{\partial\varphi}{\partial e} e \sin v_{\text{н}} \sin v_{\text{к}}\right] + \\ \left. + \frac{\partial F}{\partial e}\left[\frac{1}{\sqrt{p}}(a_1 \sin v_{\text{к}} - a_2 \sin v_{\text{н}}) + \frac{\partial\varphi}{\partial p} e \sin v_{\text{н}} \sin v_{\text{к}}\right]\right\}, \\ \nu = \frac{a_2^2 - a_1^2}{e\sqrt{p}}(a_2 \cos v_{\text{к}} - a_1 \cos v_{\text{н}}) + \frac{\partial\varphi}{\partial p} a_1 a_2 \sin(v_{\text{к}} - v_{\text{н}}) + \\ + \frac{\partial\varphi}{\partial e}(a_2 \sin v_{\text{к}} - a_1 \sin v_{\text{н}}), \\ \tau_0\left(k\mu + n_1\lambda + \frac{\partial F}{\partial t_1}\nu\right) = 0, \\ \tau_{\text{п}}(k\mu + n_2\lambda) = 0. \end{gathered}\right\} \tag{15}$$

Here, instead of the former variables μ_1 and λ_5, new μ and λ are introduced according to the formulas

$$\mu = \mu_1 \nu, \quad \lambda = \lambda_5 \nu.$$

The boundary conditions of (1) are much simplified. Thus, four first conditions are equal to two:

$$\begin{aligned} p &= a_1 (1 + e \cos v_н), \\ p &= a_2 (1 + e \cos v_к). \end{aligned} \tag{16}$$

The following four equations produce the same thing.

Getting rid of ω_1 and ω_2 and considering that $t_0 = \frac{1}{2}(t_1 + t_4)$, we find that

$$\begin{aligned} u_{10} - u_{20} + n_1 t_1 - n_2 t_2 + v_к - v_н + 2\pi l_1 &= 0, \\ u_{20} - u_{10} + n_1 t_1 - n_2 t_2 + v_к - v_н + 2\pi l_2 &= 0. \end{aligned} \tag{17}$$

Hence

$$u_{10} - u_{20} = \pi (l_2 - l_1),$$

i.e., at the initial moment in time, the points on their orbits and the center of gravity are located on a single line. Both equations in (17) are the same equation which can be given the form

$$n_2 t_2 - n_1 t_1 = v_к - v_н + l\pi, \quad l = l_1 + l_2. \tag{18}$$

The two last equations from the group of equations (1) are equivalent to the following:

$$k(t_2 - t_1) = \varphi(p, e, v_н, v_к). \tag{19}$$

We will call the Homann flyback a flyback whose orbits "there" and "back" are Homann semi-ellipses. /47

Let

$$
\begin{aligned}
T_0 &= T_{\text{г}0}(1-\varkappa_0),\\
T_{\text{п}} &= T_{\text{гп}}(1-\varkappa_{\text{п}}),
\end{aligned}
$$

where T_{h0}--the duration of Homann flyback, T_{hs}--time of stay in orbit of destination for Homann flyback, κ_0, κ_s--small parameters, smaller units in terms of the modulus. Now time limitations (3) can be given the following form:

$$
\begin{aligned}
\tau_0^2 - 2t_1 &= T_{\text{г}0}(1-\varkappa_0),\\
\tau_{\text{п}}^2 + 2t_2 &= -T_{\text{гп}}(1-\varkappa_{\text{п}}).
\end{aligned}
\tag{20}
$$

Consequently, a system is derived of 11 equations (15), (16), (18), (19), (20) with 11 unknowns p, e, v_i, v_u, t_1, t_2, τ_0, τ_s, λ, μ, ν. The last three unknowns are auxiliary and we will not derive them.

Four cases of equality to zero of the left parts of the two last equations (15) are possible.

The first case: $\tau_0 = \tau_s = 0$. Equations (20) permit us to find t_1 and t_2 by the formulas

$$
t_1 = -\frac{1}{2}T_{\text{г}0}(1-\varkappa_0), \quad t_2 = -\frac{1}{2}T_{\text{гп}}(1-\varkappa_{\text{п}}), \tag{21}
$$

after which equations (16), (18), (19) form a system with the unknowns p, e, v_i, v_u. Its solution does not depend on the form of function F.

The second case: $\tau_0 = k\mu + n_2\lambda = 0$. The orbit can be optimal if as a result of the solution we find that

$$\boxed{2t_2 + T_{\text{гп}}(1-\varkappa_{\text{п}}) \leqslant 0.} \tag{22}$$

The third case: $\tau_s = k\mu + n_1\lambda + \frac{\partial F}{\partial t_1}\nu = 0$, can produce an optimal orbit if

$$\boxed{2t_1 + T_{\text{г}0}(1-\varkappa_0) \geqslant 0.} \tag{23}$$

And finally, the fourth case: $k\mu + n_1\lambda + \frac{\partial F}{\partial t_1}\nu = k\mu + n_2\lambda = 0$, so that the orbit is optimal, we must have simultaneous fulfillment of inequalities (22) and (23).

In solution of the problem we will retain the following analytic form of function F:

$$\boxed{F[1+\varkappa(t_4-t_1)]\cdot f(\Delta V_1, \Delta V_2, \Delta V_3, \Delta V_4),} \tag{24}$$

where the quantity $\kappa(t_4 - t_1)$ is small in comparison with one and thus, can be considered a small parameter.

If $\kappa = \kappa_0 = \kappa_s = 0$, all four cases merge into one. The solution of the system is a Homann flyback. Let us write this solution: /48

$$\boxed{\begin{gathered}
p_0 = \frac{a_1a_2}{a_0}, \quad e_0 = \pm\frac{a_2-a_1}{a_2+a_1}, \quad a_0 = \frac{1}{2}(a_1+a_2), \\
n_0 = k a_0^{-3/2}, \quad v_{\text{н}0} = \begin{cases} 0 \\ \pi \end{cases}, \quad v_{\text{к}0} = \begin{cases} \pi \\ 2\pi \end{cases}, \\
\tau_{00} = \tau_{\text{п}0} = \mu_0 = \lambda_0 = 0, \quad \nu_0 = -8\frac{a_0^3}{\sqrt{p_0}}, \\
T_{\text{г}0} = \frac{2\pi}{n_1-n_2}\left(1+l-\frac{n_2}{n_0}\right), \quad T_{\text{гп}} = T_{\text{г}0} - \frac{2\pi}{n_0}, \\
l = \begin{cases} E\left(\frac{n_1-n_0}{n_0}\right)+1, \\ -1, \end{cases} \\
t_{10} = -\frac{1}{2}T_{\text{г}0}, \quad t_{20} = -\frac{1}{2}T_{\text{гп}}, \\
\Delta V_{10} = \mp\frac{k}{\sqrt{a_1}}\left(1-\sqrt{1\pm e_0}\right), \\
\Delta V_{20} = \pm\frac{k}{\sqrt{a_2}}\left(1-\sqrt{1\mp e_0}\right).
\end{gathered}} \tag{25}$$

Note:

1. In this solution are contained additional quantities: a_0--semimajor axis of the flyback ellipse, n_0--average motion along this ellipse, ΔV_{10}, ΔV_{20}--first and second pulses.

2. Here and henceforth, where there are double values or double signs, the top value and top sign will correspond to flyback from an internal orbit to the outside and back; the lower value and sign will correspond to flyback from the outside orbit to the inside and back.

3. Examples: As examples, let us cite both types of Homann flyback.

a) Homann flyback Earth--Mars--Earth (top values and top signs). In the capacity of a_1, a_2 we will select parameters of the planetary orbits, and as mean motion n_1 and n_2--the mean motion corresponding to these parameters, namely $n_1 = ka_1^{-3/2}$, $n_2 = ka_2^{-3/2}$. Then

$$a_1=0{,}999720, \quad n_1=0{,}0172093,$$
$$a_2=1{,}51040, \quad n_2=0{,}00926707.$$

For Homann flyback we find the following values:

$$p_0=1{,}20311, \quad e_0=0{,}203450, \quad a_0=1{,}25506,$$
$$v_{н0}=0, \quad v_{к0}=\pi, \quad n_0=0{,}0122344,$$
$$\Delta V_{10}=0{,}00166916=2{,}89012 \text{ км/сек},$$
$$\Delta V_{20}=0{,}00150473=2{,}60541 \text{ км/сек},$$

$$T_{г0}=982^{d}{,}991, \quad T_{гм}=469^{d}{,}423, \quad l=1,$$
$$t_{40}=-t_{10}=491^{d}{,}496, \quad t_{30}=-t_{20}=234^{d}{,}712.$$

$$\delta e = \frac{e_0}{4}\left(\frac{a_1}{a_0}\,\delta v_{\text{н}}^2 + \frac{a_2}{a_0}\,\delta v_{\text{к}}^2\right), \tag{26}$$

and equations (18) and (19) can be written as

$$\begin{aligned} \delta t_1 &= \frac{\delta v_{\text{к}} - \delta v_{\text{н}} - n_2 k^{-1}\delta\varphi}{n_2 - n_1}, \\ \delta t_2 &= \frac{\delta v_{\text{к}} - \delta v_{\text{н}} - n_1 k^{-1}\delta\varphi}{n_2 - n_1}. \end{aligned} \tag{27}$$

Using formulas (2) and (26), we find that

$$\delta\varphi = \frac{1}{\sqrt{p_0}}\left(a_2^2\delta v_{\text{к}} - a_1^2\delta v_{\text{н}}\right) \pm \frac{3\pi}{8}\,k\,\frac{e_0}{n_0}\left(\frac{a_2}{a_1}\,\delta v_{\text{к}}^2 - \frac{a_1}{a_2}\,\delta v_{\text{н}}^2\right). \tag{28}$$

In formulas (15) the partial derivatives $\frac{\partial F}{\partial p}$ and $\frac{\partial F}{\partial e}$ are multiplied by values of the first order of smallness, and thus they themselves can be calculated with consideration of the first order of small /50 values. Consequently,

$$\begin{aligned} \frac{\partial F}{\partial p} &= 2\,(1 + \varkappa T_{\text{г}0}) \sum_{i=1}^{2} \left(\frac{\partial f}{\partial\,\Delta V_i}\right)_0 \left(\frac{\partial\,\Delta V_i}{\partial p}\right)_0, \\ \frac{\partial F}{\partial e} &= 2\,(1 + \varkappa T_{\text{г}0}) \sum_{i=1}^{2} \left(\frac{\partial f}{\partial\,\Delta V_i}\right)_0 \left(\frac{\partial\,\Delta V_i}{\partial e}\right)_0. \end{aligned} \tag{29}$$

In turn, the partial derivative $\frac{\partial F}{\partial t_1}$ is sufficiently represented as

$$\frac{\partial F}{\partial t_1} = -\varkappa f_0. \tag{30}$$

Now, using formulas (15), (26), (29) and (30), we can derive

$$\begin{aligned}
\delta\mu &= (1+\varkappa T_{\text{г}0}) \sum_{i=1}^{2} \left(\frac{\partial f}{\partial \Delta V_i}\right)_0 \cdot \left\{ \left[\pm \left(\frac{\partial \Delta V_i}{\partial e}\right)_0 - a_2 \left(\frac{\partial \Delta V_i}{\partial p}\right)_0 \right] a_1 \delta v_{\text{н}} + \right. \\
&\quad \left. + \left[\pm \left(\frac{\partial \Delta v_i}{\partial e}\right)_0 + a_1 \left(\frac{\partial \Delta V_i}{\partial p}\right)_0 \right] a_2\, \delta v_{\text{к}} \right\}, \\
\delta\lambda &= a_1 a_2 (1+\varkappa T_{\text{г}0}) \sum_{i=1}^{2} \left(\frac{\partial f}{\partial \Delta V_i}\right)_0 \frac{1}{\sqrt{p_0}} \left[\left(\frac{\partial \Delta V_i}{\partial p}\right)_0 (a_2^2 \delta v_{\text{н}} - a_1^2 \delta v_{\text{к}}) \mp \right. \\
&\quad \left. \mp \left(\frac{\partial \Delta V_i}{\partial e}\right)_0 (a_1 \delta v_{\text{к}} + a_2 \delta v_{\text{н}}) \right] - \\
&\quad - a_1 a_2 e_0 \sum_{j=1}^{2} \left(\frac{\partial f}{\partial \Delta V_i}\right)_0 \cdot \left[\left(\frac{\partial \Delta V_i}{\partial p}\right)_0 \left(\frac{\partial \varphi}{\partial e}\right)_0 + \left(\frac{\partial \Delta V_i}{\partial e}\right)_0 \left(\frac{\partial \varphi}{\partial p}\right)_0 \right] \delta v_{\text{н}} \delta v_{\text{к}}, \\
\delta \left(\frac{\partial F}{\partial t_1} \nu\right) &= 8 \frac{a_0^3}{\sqrt{p_0}} f_0 \varkappa + a_1 a_2 \left(\frac{\partial \varphi}{\partial p}\right)_0 f_0 (\delta v_{\text{н}} - \delta v_{\text{к}}) \varkappa \pm \\
&\quad \pm \left(\frac{\partial \varphi}{\partial e}\right)_0 f_0 (a_1 \delta v_{\text{н}} + a_2\, \delta v_{\text{к}}) \varkappa.
\end{aligned} \tag{31}$$

Case I. $\tau_0 = \tau_s = 0$.

If time limitations of flyback and designation orbital stay time are rigid, it may turn out that the optimum is flyback where $\tau_0 = \tau_s = 0$.

Formulas (21) yield

$$\delta t_1 = \frac{1}{2} T_{\text{г}0} \varkappa_0, \quad \delta t_2 = \frac{1}{2} T_{\text{гп}} \varkappa_{\text{п}}. \tag{32}$$

Now, equations (27) may be written as

$$\begin{aligned}
M_{\text{н}}\, \delta v_{\text{н}} + M_{\text{к}}\, \delta v_{\text{к}} &= \frac{n_2 - n_1}{2} T_{\text{г}0} \varkappa_0 + \frac{n_2}{k} (\delta\varphi)', \\
N_{\text{н}} \delta v_{\text{н}} - N_{\text{к}} \delta v_{\text{к}} &= \frac{n_2 - n_1}{2} T_{\text{гп}} \varkappa_{\text{п}} + \frac{n_1}{k} (\delta\varphi)',
\end{aligned} \tag{33}$$

where /51

$$M_{\text{н}} = \frac{(1 \mp e_0)^{3/2} - (1 \pm e_0)^2}{(1 \pm e_0)^2}, \quad M_{\text{к}} = \frac{\sqrt{1 \mp e_0} - 1}{\sqrt{1 \mp e_0}}, \tag{34}$$

$$N_{\text{н}}=\frac{1-\sqrt{1\pm e_0}}{\sqrt{1\pm e_0}}, \quad N_{\text{к}}=\frac{(1\mp e_0)^2-(1\pm e_0)^{3/2}}{(1\mp e_0)^2}, \tag{34}$$

while

$$(\delta\varphi)'=\delta\varphi-\frac{1}{\sqrt{p_0}}\left(a_2^2\delta v_{\text{к}}-a_1^2\delta v_{\text{н}}\right). \tag{35}$$

Let us designate the determinant of the system by D_1. We can derive that

$$D_1=\pm\frac{4e_0}{(1-e_0^2)^2}\left[(1\pm e_0)^{3/2}-(1\mp e_0)^{3/2}\right]. \tag{36}$$

Let us present the correction of any quantity y as

$$\delta y=\delta_1 y+\delta_2 y+\ldots$$

and we will equate identical orders of small quantities of the right and left in equations where these corrections appear. Then for the i^{th} order of magnitude δv_i and δv_u, we derive the system

$$\begin{aligned} M_{\text{н}}\delta_i v_{\text{н}}+M_{\text{к}}\delta_i v_{\text{к}}&=L_1^{(i)},\\ N_{\text{н}}\delta_i v_{\text{н}}+N_{\text{к}}\delta_i v_{\text{к}}&=L_2^{(i)}, \end{aligned}$$

where $L_1^{(i)}$, $L_2^{(i)}$ are the i^{th} order of corresponding right sides.

The solution of this system will be

$$\begin{aligned} \delta_i v_{\text{н}}&=\frac{N_{\text{к}}}{D_1}L_1^{(i)}-\frac{M_{\text{к}}}{D_1}L_2^{(i)},\\ \delta_i v_{\text{к}}&=-\frac{N_{\text{н}}}{D_1}L_1^{(i)}+\frac{M_{\text{н}}}{D_1}L_2^{(i)}. \end{aligned} \tag{37}$$

The first order is derived under the condition that

$$L_1^{(1)}=\frac{n_2-n_1}{2}T_{\text{г}0}\varkappa_0, \quad L_2^{(1)}=\frac{n_2-n_1}{2}T_{\text{гп}}\varkappa_{\text{п}}.$$

From formula (26) we see that for all cases of

$$\delta_1 p = \delta_1 e = 0,$$

and the first order of $\delta_1 v_i$ and $\delta_1 v_u$ permits us to calculate $\delta_2 p$ and $\delta_2 e$ according to the formulas

$$\begin{aligned} \delta_2 p &= \pm \frac{p_0 e_0}{4} (\delta_1 v_{\text{к}}^2 - \delta_1 v_{\text{н}}^2), \\ \delta_2 e &= \frac{e_0}{4} \left(\frac{a_1}{a_0} \delta_1 v_{\text{н}}^2 + \frac{a_2}{a_0} \delta_1 v_{\text{к}}^2 \right). \end{aligned} \tag{38}$$

The second and higher orders can be calculated according to simpler formulas, since the right sides of the equations for these orders acquire the following form: /52

$$L_1^{(l)} = n_2 L^{(l)}, \qquad L_2^{(l)} = n_1 L^{(l)}.$$

Therefore

$$\begin{aligned} \delta_l v_{\text{н}} &= (n_2 N_{\text{к}} - n_1 M_{\text{к}}) D_1^{-1} L^{(l)}, \\ \delta_l v_{\text{к}} &= (-n_2 N_{\text{н}} + n_1 M_{\text{н}}) D_1^{-1} L^{(l)}. \end{aligned}$$

Calculations yield

$$\frac{n_2 N_{\text{к}} - n_1 M_{\text{к}}}{D_1} = \frac{-n_2 N_{\text{н}} + n_1 M_{\text{н}}}{D_1} = \mp \frac{n_0}{4 e_0} \sqrt{1 - e_0^2}.$$

Therefore, for all subsequent orders we have

$$\delta_l v_{\text{н}} = \delta_l v_{\text{к}} = \mp \frac{n_0}{4 e_0} \sqrt{1 - e_0^2}\, L^{(l)}. \tag{39}$$

For the second order

$$L^{(2)} = \pm \frac{3\pi}{8} \cdot \frac{e_0}{n_0} \left(\frac{a_2}{a_1} \delta_1 v_к^2 - \frac{a_1}{a_2} \delta_1 v_н^2 \right). \tag{40}$$

Generally, $L^{(i)}$ is the sum of terms of the i^{th} order of magnitude of $k^{-1}(\delta\phi)'$, $\delta_2 v_i$, $\delta_2 v_u$ permit us to calculate $\delta_3 p$ and $\delta_3 e$ by the formulas

$$\delta_3 p = \pm \frac{p_0 e_0}{2} (\delta_1 v_к \delta_2 v_к - \delta_1 v_н \delta_2 v_н),$$

$$\delta_3 e = \frac{e_0}{2} \left(\frac{a_1}{a_0} \delta_1 v_н \delta_2 v_н + \frac{a_2}{a_0} \delta_1 v_к \delta_2 v_к \right). \tag{41}$$

<u>Case II.</u> $\tau_0 = k\mu + n_2\lambda = 0.$

The first equation (20) yields

$$\delta t_1 = \frac{1}{2} T_{r0} x_0. \tag{42}$$

Then the first equation (27) and equation

$$k\mu + n_2\lambda = 0 \tag{43}$$

form a system with respect to δv_i and δv_u. Presenting in equations (27) and (43) a decomposition of (28) and (31) for the i^{th} order of small quantities we find that

$$\begin{aligned} M_н \delta_i v_н + M_к \delta_i v_к &= L_1^{(i)}, \\ Y_н \delta_i v_н + Y_к \delta_i v_к &= x G^{(i)}, \end{aligned} \tag{44}$$

where

$$Y_н = (1 \mp e_0) M_к A, \quad Y_к = -(1 \pm e_0) M_н B,$$

$$A = \frac{(3 \mp e_0)(1 \pm e_0) + 4\sqrt{1 \pm e_0}}{1 - e_0^2} \left(\frac{\partial f}{\partial \Delta V_1} \right)_0 + \left(\frac{\partial f}{\partial \Delta V_2} \right)_0,$$

$$B = \left(\frac{\partial f}{\partial \Delta V_1} \right)_0 + \frac{(3 \pm e_0)(1 \mp e_0) + 4\sqrt{1 \mp e_0}}{1 - e_0^2} \left(\frac{\partial f}{\partial \Delta V_2} \right)_0,$$

$$x = \pm \frac{2(1-e_0^2)}{k^2 \sqrt{p_0}}. \tag{45}$$

The first equation of system (44) completely coincides with the first equation of system (33) written for the i^{th} order of magnitudes δv_i and δv_u. /53

In $G^{(i)}$ are gathered all terms of the i^{th} order of expression $k\mu + n_2\lambda$, taken with inverted signs, plus terms containing $\delta_i v_i$ and $\delta_i v_u$, from which is formed the left side of the equation.

Let us assume that

$$\left(\frac{\partial f}{\partial \Delta V_1}\right)_0 > 0, \quad \left(\frac{\partial f}{\partial \Delta V_2}\right)_0 > 0, \tag{46}$$

then the following inequalities are valid:

$$M_н \lesseqgtr 0, \quad M_к \lesseqgtr 0, \quad Y_н \lesseqgtr 0, \quad Y_к \lesseqgtr 0, \quad D_2 < 0, \tag{47}$$

where D_2 is the determinant of system (44). Let us remember that the top signs of the inequalities are taken for flyback from the inside orbit to the outside and back. Let us introduce the notations

$$\begin{aligned} x_н &= x M_н D_2^{-1}, & x_к &= x M_к D_2^{-1}, \\ y_н &= \pm Y_н D_2^{-1}, & x_к &= \mp Y_к D_2^{-1}. \end{aligned} \tag{48}$$

Now the solution of system (44) can be written as follows:

$$\begin{aligned} \delta_i v_н &= \mp y_к L_1^{(i)} - x_к G^{(i)}, \\ \delta_i v_к &= \mp y_н L_1^{(i)} + x_н G^{(i)}, \end{aligned} \tag{49}$$

where $x_i > 0$, $x_u > 0$, $y_i > 0$, $y_u > 0$. The first order we derive assuming that

$$L_1^{(1)} = \frac{n_2 - n_1}{2} T_{\text{г}0} \varkappa_0, \quad G^{(1)} = 0.$$

Because $L_1^{(1)} = \mp |L_1^{(1)}|$ when $\chi_0 > 0$, then $\delta_1 v_i > 0$, $\delta_1 v_u > 0$. When $\chi_0 < 0$ we find that $\delta_1 v_i < 0$, $\delta_1 v_u < 0$. The second formula (27) produces

$$\delta_1 t_2 = \frac{1}{(n_2 - n_1)} (N_{\text{н}} \delta_1 v_{\text{н}} + N_{\text{к}} \delta_1 v_{\text{к}}). \tag{50}$$

When $\chi_0 > 0$ and $\delta_1 t_2 > 0$; when $\chi_0 < 0$, $\delta_1 t_2 < 0$. At small χ_0, along with reducing the total time of flight, the stay in destination orbit also decreases and vice versa.

The second order is derived under the condition that

$$\begin{aligned} L_1^{(2)} &= \pm \frac{3\pi}{8} \cdot \frac{n_2}{n_0} e_0 \left(\frac{a_2}{a_1} \delta_1 v_{\text{к}}^2 - \frac{a_1}{a_2} \delta_1 v_{\text{н}}^2 \right), \\ G^{(2)} &= \frac{3\pi}{2} k^3 \frac{n_2}{n_0} \frac{e_0^2}{(1 - e_0^2)^2} \left[\frac{2(1 - e_0^2) - (1 \pm e_0)^{3/2}}{\Delta V_{10}} \left(\frac{\partial f}{\partial \Delta V_1} \right)_0 + \right. \\ &\left. + \frac{2(1 - e_0^2) - (1 \mp e_0)^{3/2}}{\Delta V_{20}} \left(\frac{\partial f}{\partial \Delta V_2} \right)_0 \right] \delta_1 v_{\text{н}} \delta_1 v_{\text{к}}. \end{aligned} \tag{51}$$

For $\delta_2 t_2$ from (27) we derive

$$\delta_2 t_2 = \frac{1}{n_2 - n_1} (N_{\text{н}} \delta_2 v_{\text{н}} + N_{\text{к}} \delta_2 v_{\text{к}}) - \frac{n_1}{n_2} \cdot \frac{L_1^{(2)}}{n_2 - n_1}. \tag{52}$$

This case can yield an optimal orbit if /54

$$\delta t_2 \leqslant \frac{1}{2} T_{\text{гл}} \varkappa_{\text{п}}, \tag{53}$$

where $\delta t_2 = \delta_1 t_2 + \delta\delta_2 t_2$.

Corrections for the parameter and eccentricity are calculated for all cases using formulas (38) and (41).

<u>Case III</u>. $k\mu + n_1\lambda + \frac{\partial F}{\partial t_1}\nu = \tau_s = 0.$

This case is completely analogous to the second, if everywhere is replaced the first equation of (20) with the second and on the other hand, equation (43) by the equation

$$k\mu + n_1\lambda + \frac{\partial F}{\partial t_1}\nu = 0. \tag{54}$$

Consequently, we find that

$$\delta t_2 = \frac{1}{2} T_{\text{гп}} \varkappa_{\text{п}} \tag{55}$$

and the system

$$\begin{aligned} N_{\text{н}}\delta_l v_{\text{н}} + N_{\text{к}}\delta_l v_{\text{к}} &= L_2^{(l)}, \\ Z_{\text{н}}\delta_l v_{\text{н}} + Z_{\text{к}}\delta_l v_{\text{к}} &= xH^{(l)}, \end{aligned} \tag{56}$$

where

$$Z_{\text{н}} = (1 \mp e_0) N_{\text{к}} A, \qquad Z_{\text{к}} = -(1 \pm e_0) N_{\text{н}} B, \tag{57}$$

and $H^{(1)}$ is derived by analogy with $G^{(1)}$ from the left side of equation (54). Taking into account the assumption of (46), we can show that

$$N_{\text{н}} \lesseqgtr 0, \quad N_{\text{к}} \lesseqgtr 0, \quad Z_{\text{н}} \gtreqless 0, \quad Z_{\text{к}} \gtreqless 0, \quad D_3 < 0, \tag{58}$$

where D_3 is the determinant of system (56).

If we designate that

$$\begin{aligned} w_{\text{н}} &= xN_{\text{н}}D_3^{-1}, & w_{\text{к}} &= xN_{\text{к}}D_3^{-1}, \\ z_{\text{н}} &= \pm Z_{\text{н}}D_3^{-1}, & z_{\text{к}} &= \mp Z_{\text{к}}D_3^{-1}, \end{aligned} \tag{59}$$

then the solution of system (56) will appear as

$$\begin{aligned} \delta_i v_{н} &= \mp z_{к} L_2^{(i)} - w_{к} H^{(i)}, \\ \delta_i v_{к} &= \mp z_{н} L_2^{(i)} + w_{н} H^{(i)}, \end{aligned} \tag{60}$$

wherein

$$z_{н}>0, \quad z_{к}>0, \quad w_{н}>0, \quad w_{к}>0.$$

To derive the first order we assume that

$$L_2^{(1)} = \frac{n_2-n_1}{2} T_{гп} \varkappa_{п}, \quad H^{(1)} = -8 \frac{a_0^3}{\sqrt{p_0}} f_0 \varkappa. \tag{61}$$

If $f_0\chi > 0$, then when $\chi_s > 0$ we yield $\delta_1 v_i > 0$, $\chi_s < 0 - -\delta_1 v_u < 0$,

$$\delta_1 t_1 = \frac{1}{n_2-n_1} (M_{н} \delta_1 v_{н} + M_{к} \delta_1 v_{к}). \tag{62}$$

Substituting in (62) $\delta_1 v_i$, $\delta_1 v_u$ expressed with the aid of formulas /55
(6), we can be sure that when $\chi_s > 0$ and $\delta_1 t_1 > 0$. That is, the presence of an internal reserve of mass and the reduction in time of orbital stay (destination orbit) reduce the total duration of the flight. To derive the second order we assume that

$$\begin{aligned} L_2^{(2)} &= \pm \frac{3\pi}{8} \cdot \frac{n_1}{n_0} e_0 \left(\frac{a_2}{a_1} \delta_1 v_{к}^2 - \frac{a_1}{a_2} \delta_1 v_{н}^2 \right), \\ H^{(2)} &= \frac{n_1}{n_2} G^{(2)} + \left[8 \frac{a_0^3}{\sqrt{p_0}} T_{г0} \varkappa_0 + \frac{3}{2} \cdot \frac{k\pi}{n_0 p_0} (a_1^2 \delta_1 v_{н} - a_2^2 \delta_1 v_{к}) \right] f_0 \varkappa. \end{aligned} \tag{63}$$

The first equation of (27) yields

$$\delta_2 t_1 = \frac{1}{n_2-n_1} (M_{н} \delta_2 v_{н} + M_{к} \delta_2 v_{к}) - \frac{n_2}{n_1} \cdot \frac{L_2^{(2)}}{n_2-n_1}. \tag{64}$$

This case can be an optimal orbit if the inequality is fulfilled

$$\delta t_1 \geqslant \frac{1}{2} T_{r0} \varkappa_{\mathrm{J}}, \tag{65}$$

where $\delta t_1^{\circ} = \delta_1 t_1 + \delta_2 t_1$.

<u>Case IV</u>.

Necessary conditions of the minimum are

$$\begin{aligned} k\mu + n_1\lambda + \frac{\partial F}{\partial t_1}\nu = 0, \\ k\mu + n_2\lambda = 0 \end{aligned} \tag{66}$$

can be represented in the form of a system of equations relative to $\delta_i v_i$, $\delta_i v_u$. It appears that the system has the form

$$\begin{aligned} Y_{\text{н}}\delta_i v_{\text{н}} + Y_{\text{к}}\delta_i v_{\text{к}} = xG^{(i)}, \\ Z_{\text{н}}\delta_i v_{\text{н}} + Z_{\text{к}}\delta_i v_{\text{к}} = xH^{(i)}. \end{aligned} \tag{67}$$

Here all notations are as before.

For the determinant of this system D_4, the calculations yield the following expression:

$$D_4 = (1 - e_0^2)\, ABD_1. \tag{68}$$

Considering (46), we yield D_4 0. Let

$$\begin{aligned} \eta_{\text{н}} &= -xY_{\text{н}}D_4^{-1}, & \eta_{\text{к}} &= xY_{\text{к}}D_4^{-1}, \\ \zeta_{\text{н}} &= -xZ_{\text{н}}D_4^{-1}, & \zeta_{\text{к}} &= xZ_{\text{к}}D_4^{-1}, \end{aligned} \tag{69}$$

then the solution of system (67) will be

$$\left.\begin{aligned}\delta_i v_н &= \zeta_к G^{(i)} - \eta_к H^{(i)},\\ \delta_i v_к &= \zeta_н G^{(i)} - \eta_н H^{(i)};\end{aligned}\right\} \tag{70}$$

where $\eta_i > 0$, $\eta_u > 0$, $\zeta_i > 0$, $\zeta_u > 0$,

First Order /56

$$G^{(1)}=0, \quad H^{(1)} = -8\frac{a_0^3}{\sqrt{p_0}} f_0 \varkappa. \tag{71}$$

If $f_0\kappa > 0$, then $\delta_1 v_i > 0$, $\delta_1 v_u > 0$.

We derived from formula (27) that

$$\begin{aligned}\delta_1 t_1 &= (n_2 - n_1)^{-1}(M_н \delta_1 v_н + M_к \delta_1 v_к),\\ \delta_1 t_2 &= (n_2 - n_1)^{-1}(N_н \delta_1 v_н + N_к \delta_1 v_к).\end{aligned} \tag{72}$$

With these same assumptions $\delta_1 t_1 > 0$, $\delta_1 t_2 > 0$; consequently, there is a reduction in both the total duration of the flight and the destination orbit stay time.

Second Order

$G^{(2)}$ and $H^{(2)}$ are calculated according to formulas (51) and (63). For $\delta_2 t_1$ and $\delta_2 t_2$ we find that

$$\begin{aligned}\delta_2 t_1 &= (n_2 - n_1)^{-1}(M_н \delta_2 v_н + M_к \delta_2 v_к - L_1^{(2)}),\\ \delta_2 t_2 &= (n_2 - n_1)^{-1}(N_н \delta_2 v_н + N_к \delta_2 v_к - L_2^{(2)}).\end{aligned} \tag{73}$$

This case yields an optimal orbit if there is fulfillment simultaneously of two inequalities:

$$\left.\begin{aligned}\delta t_1 &\geqslant \frac{1}{2} T_{г0} \varkappa_0,\\ \delta t_2 &\leqslant \frac{1}{2} T_{гп} \varkappa_п.\end{aligned}\right\} \tag{74}$$

Selection of an Optimal Orbit

The optimal orbit should satisfy, above all, conditions (3) with real τ_0 and τ_s. These conditions for the three last cases are written as inequalities (53), (65), (74). The first case always yields an orbit which satisfies conditions (3), but it will not always be an optimal orbit. An additional criterion of selection is the magnitude of function F, calculated in these orbits. For this purpose, we will calculate a correction for the function F, induced by deviation of the flyback orbit from Homann flyback. Using the property of function F (5) and its form (24), we find that

$$\frac{1}{2}\delta F = \varkappa f_0\left(\frac{T_{\text{г}0}}{2} - \delta t_1\right) + \sum_{i=1}^{2}\left(\frac{\partial f}{\partial \Delta V_i}\right)_0 \delta(\Delta V_i). \tag{75}$$

Corrections for pulses can be found using formulas (4) and (26).

$$\begin{aligned} \delta(\Delta V_1) &= \mp \frac{k^2 e_0}{8\Delta V_{10} p_0}\left[(1 \mp e_0)^2 N_{\text{к}} \delta v_{\text{н}}^2 + (1 \pm e_0)^2 N_{\text{н}} \delta v_{\text{к}}^2\right], \\ \delta(\Delta V_2) &= \mp \frac{k^2 e_0}{8\Delta V_{20} p_0}\left[(1 \mp e_0)^2 M_{\text{к}} \delta v_{\text{н}}^2 + (1 \pm e_0)^2 M_{\text{н}} \delta v_{\text{к}}^2\right]. \end{aligned} \tag{76}$$

Therefore, always /57

$$\delta F > 0,$$

if $f_0\varkappa > 0$ and assumptions (46) are true.

If we are limited to the second order of small values, the correction F, calculated by formula (75), is an additional criterion for selecting the optimal orbit. The orbit for which δF is smallest is the optimal orbit.

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